Aircraft Tracking Using a Radar and the Smooth Variable Structure Filter

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1. Introduction

The smooth variable structure filter (SVSF) is a predictor-corrector method used for state and parameter estimation. This paper applies the SVSF to an air traffic control problem, whereby an aircraft is tracked based on position measurements. The results of the SVSF are qualitatively compared to those of the Extended Kalman Filter (EKF). In this case, it is shown that the SVSF yields more accurate results, and is a more robust and stable estimation method.

2. The Smooth Variable Structure Filter

The SVSF is a relatively new method used for state estimation. The model is based on and applies to smooth nonlinear dynamic systems [1]. The SVSF becomes more accurate with better defined boundaries on parameter changes and uncertainties [1]. It is a type of sliding mode estimator, whereby an algorithm with switching present is used to converge on the true (or accurate) values of states.

The basic concept of the SVSF is shown in Figure 1. Assume that the solid line is a trajectory of some state (amplitude versus time). Some initial value is selected for the estimated state (or states). The estimated state is then pushed towards the true value into a neighbouring region referred to as the existence subspace. Once the estimate enters the existence subspace, it is confined to it and is forced to switch back and forth across the true state trajectory [1]. The width of the existence subspace is dependent on the disturbances and uncertainties present in the system, and it also impacts the estimation accuracy [1].

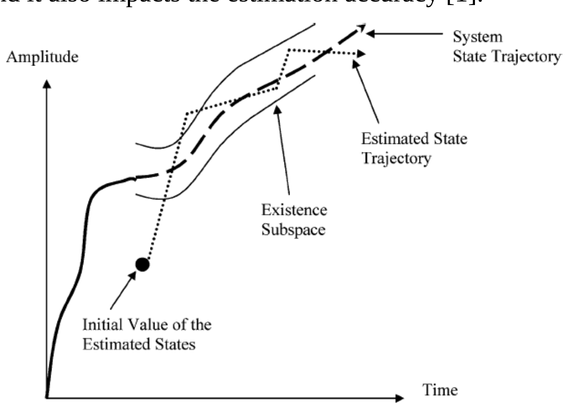

Fig. 1: SVSF state estimation concept [1].

The SVSF can be applied to both linear and nonlinear systems. It is a "predictor-corrector method that uses an internal model to predict an a priori estimate of the states" [1]. An a priori estimate is corrected into a posteriori state estimate by using a corrective term. The SVSF estimation process (which is iterative) may be summarized as follows:

1. The a priori estimate is predicted by using the estimated model of the system, which is found using a previous a posteriori state estimate (as per the following equations). Note that initial conditions would be used to start this process.

$$\hat{x}_{k+1|k} = \hat{f}\left(\hat{x}_{k|k}, u_k\right) \tag{1}$$

$$\hat{z}_{k+1|k} = \hat{H}\hat{x}_{k+1|k} \tag{2}$$

2. A corrective term is calculated as a function of the error.

$$K_k = \hat{H}^{-1}\left|\left(\left|e_{z_{k|k-1}}\right|_{ABS} + \gamma\left|e_{z_{k-1|k-1}}\right|_{ABS}\right)\right|_{ABS} \circ sat\left(e_{z_{k|k-1}}, \Psi\right) \tag{3}$$

Where H is the output matrix, $e_{z,k|k-1}$ and $e_{z,k-1|k-1}$ are the a priori and a posteriori errors, γ is some constant diagonal gain matrix, Ψ is the subspace thickness, and *sat* is the saturation function.

3. Based on the corrective term found in the previous step, the a priori state estimate is used to create an a posteriori state estimate.

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \tag{4}$$

The above steps are repeated until convergence is obtained. This occurs when "the magnitude of the a posteriori estimation error is reduced in time until the existence boundary layer is reached" [1]. Once this occurs, the state will remain within the existence layer [1].

3. Air Traffic Control Problem

The following target tracking problem is based on that found in section 11.7 of [2]. Suppose that a radar is stationed at the origin (0 m, 0 m) and provides direct position only measurements. A target trajectory (i.e. aircraft) is generated based on two modes of flight: uniform motion (straight and level flight with constant speed and course), and coordinated turn (turning or climbing/descending). For further information on the models, refer to [2]. The generated trajectory is shown in Figure 2. Note that the measurements are noisy and follow the trajectory.

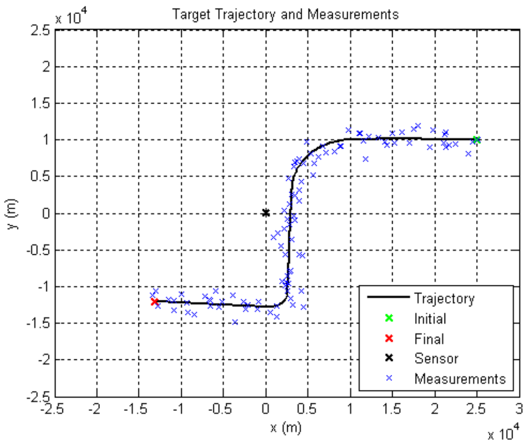

Fig. 2: Target trajectory and measurements.

4. Simulation Results

For comparisons, the above target tracking problem was solved using the EKF and the SVSF. Note that the Kalman Filter (KF) is a recursive method for detecting future state values given previous estimates and new input data, and is a popular and well-studied algorithm [2]. The EKF uses a linearized form of the nonlinear system equations in the estimation process, and is conceptually similar to the KF.

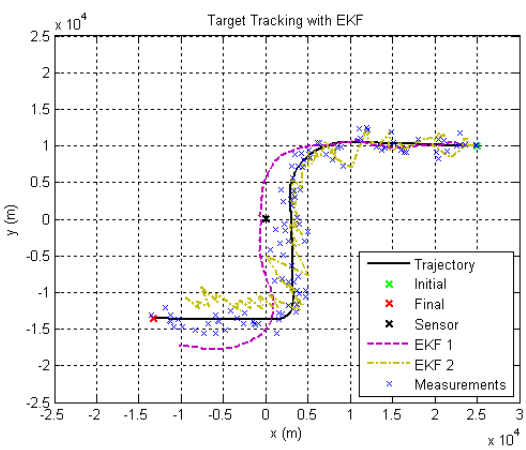

Fig. 3: Target tracking with the EKF.

Figure 3 shows the results of implementing the EKF on the problem. Note that when the first model (uniform motion) is used, the EKF does not yield valid estimates when handling the turns. However, it does start off very well. When the second model (coordinated turn) is used, the EKF yields better results. The turns are handled very well, however the uniform motion is not estimated smoothly due to the nature of the system model (coordinated turn). Inaccuracies in the estimation may be due to the linearization of the nonlinear model (coordinated turn). For example, important higher order terms may have been neglected. Note that the uniform motion model was linear.

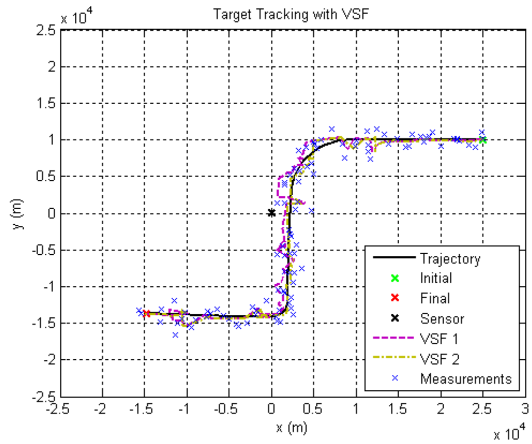

Fig. 4: Target tracking with the SVSF.

Figure 4 shows the results of implementing the SVSF on the air traffic control problem. Note that the results of the SVSF are more accurate than that of the EKF. No linearization of the nonlinear model is required when using the SVSF. Furthermore, it is interesting to note that the SVSF appeared to perform very well regardless of which model was being used. This is an important feature of the SVSF, as it demonstrates the robustness of the filter when incorrect system models are used.

Further to the above results, it was found that when incorrect initial conditions were used and modeling errors were intentionally set in the system, the EKF would fail whereas the SVSF would still function relatively well. The stability of the SVSF makes it attractive for estimation problems when not all of the system models and parameters are known accurately.

5. Conclusions

Previously [1] the SVSF had not been used on target tracking problems. The results of this short paper demonstrate that the SVSF may be a viable method for target tracking and should be studied further with other problems found throughout tracking literature. In this case, when compared to the EKF, it was shown that the SVSF yielded more accurate results, and was a more robust and stable estimation method.

References

[1] S. Habibi, *The Smooth Variable Structure Filter*, IEEE, 2007.

[2] Y. Bar-Shalom, X. Rong Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*, John Wiley and Sons, 2001.